Tests of Relativistic Gravity in 21st Century:

History, Recent Progress and Future Directions

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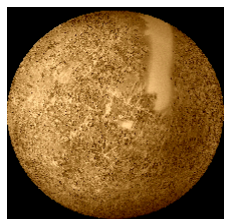
Проблемы Современной Астрометрии,
Всероссийская конференция-школа для молодых ученых
Звенигород, Россия - 22-26 октября 2007

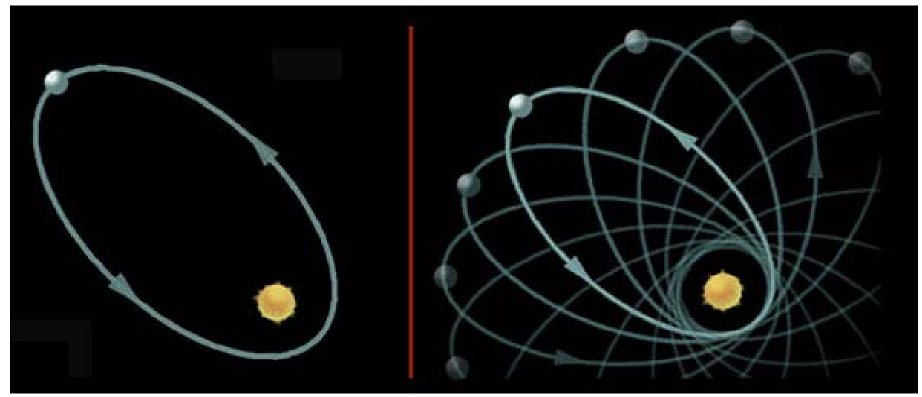
Triumph of Mathematical Astronomy in 19th Century

Discovery of Neptune: 1845

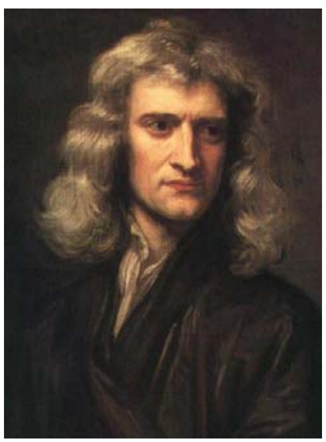
Urbain LeVerrier
(1811-1877)



- 1845: the search for Planet-X:
  - Anomaly in the Uranus' orbit → Neptune
  - Anomalous motion of Mercury → Vulcan



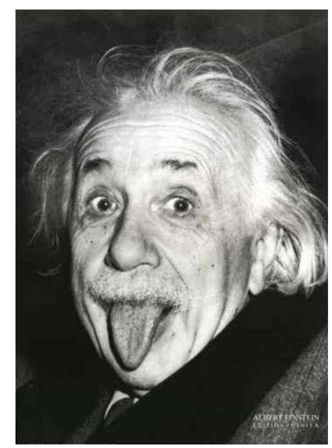
Newtonian Gravity   General Relativity



Sir Isaac Newton
(1643-1727)

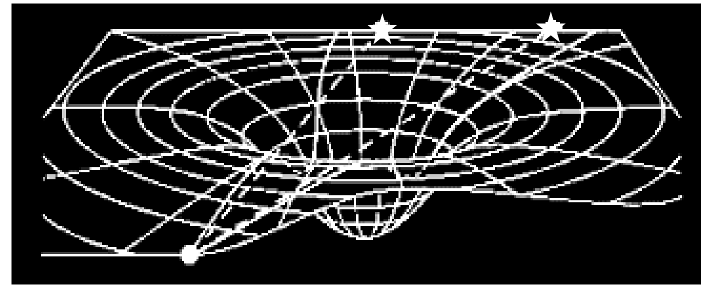
- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
  - When he got out 43 arcsec/cy – a new era just began!!

Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's' anomaly).

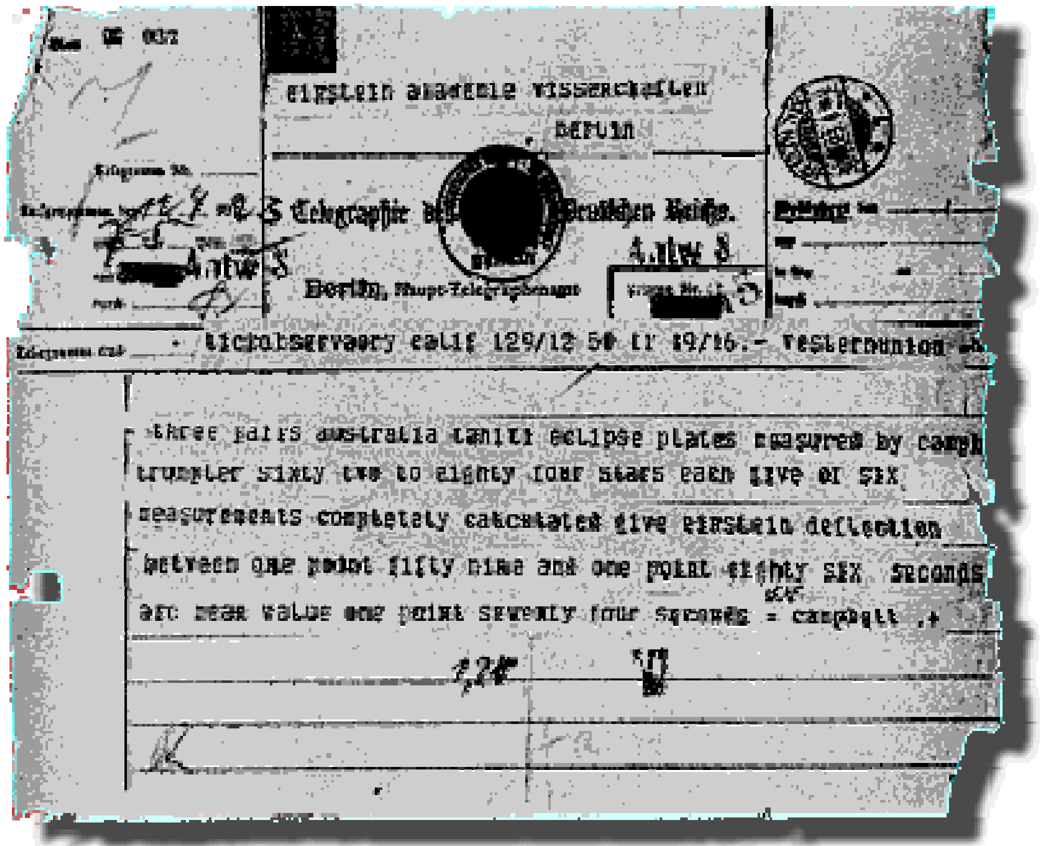


Albert Einstein
(1879-1955)

The First Test of General Theory of Relativity



Gravitational Deflection of Light: Solar Eclipse 1919



Telegraphie des Deutschen Reichs.

Berlin, Haupt-Telegraphenamt

einstein akademie wissenschaften berlin

lickobservatory calif 129/12 50 11 19/16.- westernunion

three pairs australia tanitt eclipse plates measured by campb

trumpler sixty two to eighty four stars each five or six

measurements completely calculated give einstein deflection

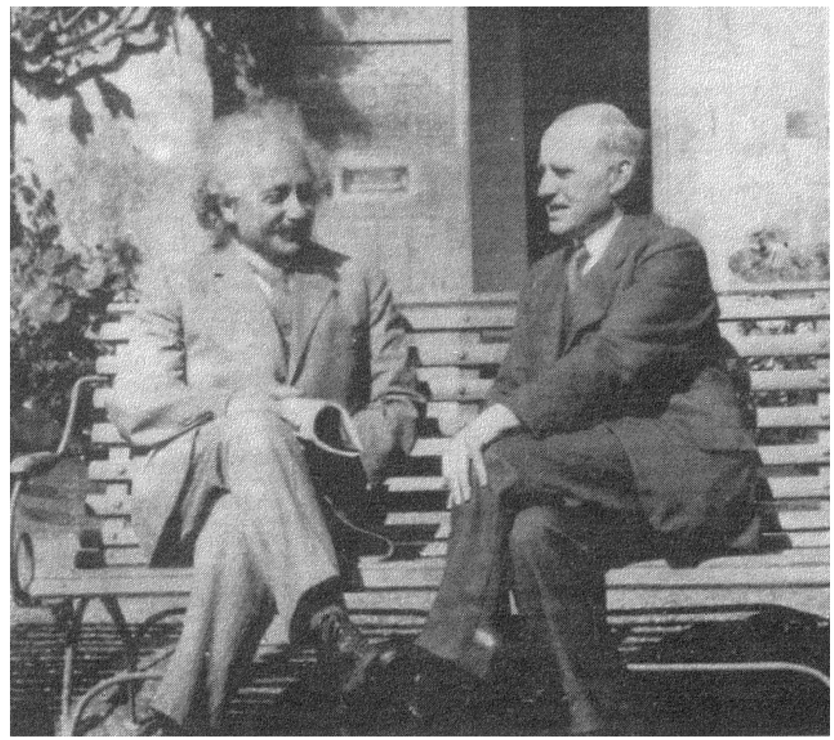
between one point fifty nine and one point eighty six seconds

arc mean value one point seventy four seconds = campbell .+

Eddington's telegram to Einstein, 1919

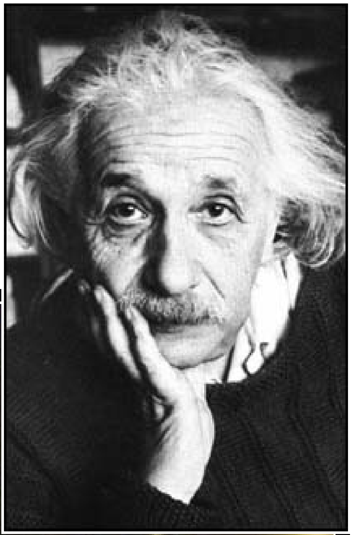
Possible outcomes in 1919:

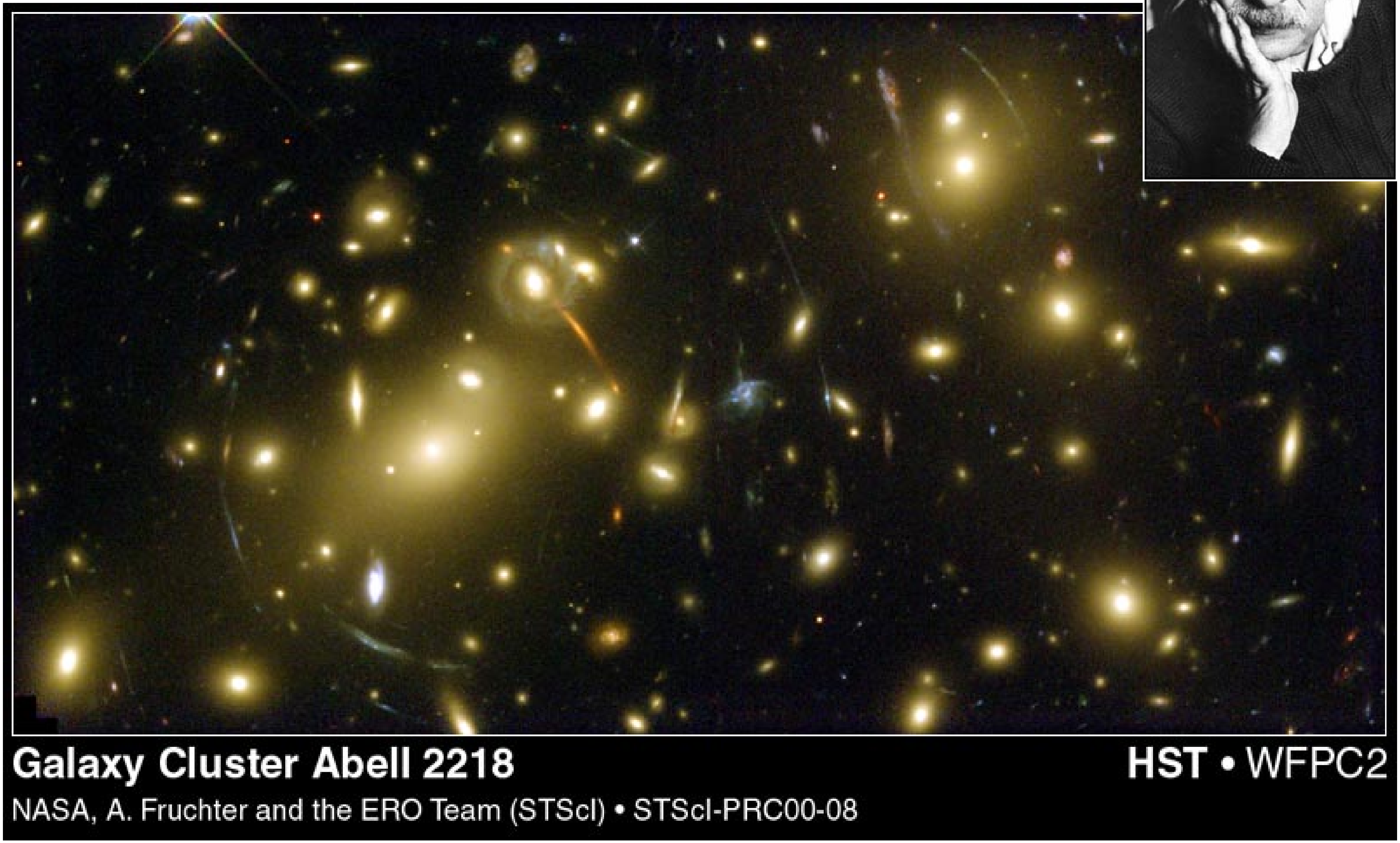
Deflection = 0;
Newton = 0.87 arcsec;
Einstein = 2 x Newton = 1.75 arcsec



Einstein and Eddington, Cambridge, 1930

Gravitational Deflection of Light is a Well-Known Effect Today





Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956
Brans & Dicke 1961	Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967	
Page & Tupper 1968	Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970	Wagoner 1970
Bollini et al. 1970	Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975
Belinfante & Swihart 1975	Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979
Coleman 1983	Logunov 1987	Hehl 1997	Overlooked (20thcentury)	

- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories are just variations of others
- Some theories were proposed in the 1910s/20s; many theories in the 1960s/70s
- Overlooked: this is not a complete list!

Theory must be:

- **Complete**: not a law, but a theory. Derive experimental results from first principles
- **Self-consistent**: get same results no matter which mathematics or models are used
- **Relativistic**: Non-gravitational laws are those of Special Relativity
- **Newtonian**: Reduces to Newton's equation in the limit of low gravity and low velocities

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that fail already

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
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- Newton (1686) - non-relativistic: implicit action at a distance - incompatible with special relativity
- Poincare (1890) and conformally flat theory of Whithrow-Morduch (1965) - incomplete: do not mesh with non-gravitational physics (Maxwell)
- Fierz & Pauli (1939) ["spin-2 field theory"] - inconsistent: field equations -> all gravitating bodies move along straight lines, equation of motion -> gravity deflects bodies
- Birkhoff (1943) - not Newtonian: demands speed of sound = speed of light.
- Milne (1948) – incomplete - no gravitational red-shift prediction
- Kustaanheimo-Nuotio (1967) – inconsistent – grav. redshift for photons, but not for light waves.

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate EEP

Newton 1686 · Poincaré 1890

Einstein 1912 · Nordstrøm 1912 · Nordstrøm 1913 · Einstein & Fokker 1914 · Einstein 1915

Whitehead 1922 · Cartan 1923 · Kaluza & Klein 1932 · Fierz & Pauli 1939 · Birkhoff 1943

Milne 1948 · Thiry 1948 · Papapetrou 1954 · Jordan 1955 · Littlewood & Bergmann 1956

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Bollini et al. 1970 · Rosen 1971 · Will & Nordtvedt 1972 · Ni 1972 · Hellings & Nordtvedt 1972

Ni 1973 · Yilmaz 1973 · Lightman & Lee 1973 · Lee, Lightman & Ni 1974 · Rosen 1975

Belinfante & Swihart 1975 · Lee et al. 1976 · Bekenstein 1977 · Barker 1978 · Rastall 1979

Coleman 1983 · Logunov 1987 · Hehl 1997 · Overlooked (20th century)

Einstein Equivalence Principle (EEP):

- Uniqueness of Free Fall
- Local Lorentz Invariance
- Local Position Invariance

Only metric theories are viable:

- Belinfante & Swihart (1975): not a metric theory
- Kaluza-Klein (1932): violates EEP
- Still too many theories around…

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate LLI

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956
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Quasi-linear theories:

- Deser & Laurent (1968), Bollini, Giambiagi & Tiomno (1970) both predict $\xi=1$
- Whitehead (1922) predicts time-dependence for ocean tides in violation of everyday experience

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate LPI

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

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Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Logunov 1987 | Hehl 1997 | Overlooked (20thcentury)

Stratified theories with time-orthogonal time slices all predict $\xi \neq 0$:

- Einstein (1912), Papapetrou (1954) (actually two theories)
- Yilmaz (1962), Whithrow & Morduch (1965)
- Page & Tupper (1968), Rosen (1971)
- Ni (1972), Coleman (1983)

Theoretical Landscape of the 20th Century:

Standard PPN Formalism

- Nordtvedt (1968),Will & Nordtvedt (1972) proposed parameterized post-Newtonian (PPN) formalism with the standard 10 Parameter-PPN Metric:

$$
\begin{aligned}
g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\
&\quad + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 \\
&\quad - (\zeta_1 - 2\xi)A - (\alpha_1 - \alpha_2 - \alpha_3)w^2U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^iV_i \\
&\quad + O(\epsilon^3), \\
g_{0i} &= -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\
&\quad - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^iU - \alpha_2 w^j U_{ij} + O(\epsilon^{5/2}), \\
g_{ij} &= (1 + 2\gamma U + O(\epsilon^2))\delta_{ij}.
\end{aligned}
$$

- Measurements of the PPN parameters done as a by-product of:
  - Spacecraft Doppler and range, planetary microwave ranging,
  - VLBI, satellite laser ranging, etc.

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that predict $\gamma = 0$ fail

Newton 1686 Poincaré 1890
Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915
Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943
Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956
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Coleman 1983 | Logunov 1987 | Hehl 1997 | Overlooked (20^{th}century)

Parameterized Post-Newtonian Formalism (PPN):

- Solar system is the main arena to test weak gravity:
- Expand the metrics; identify various potentials
- They have 10 PPN parameters in front

  $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- Calculate those parameters & Compare with experiments

[**2006**: A need for Cosmological PPN?]

Conformally-flat theories fail test of time delay and deflection of light:

- Nordstrom (1912)
- Nordstrom (1913)
- Einstein & Fokker (1914)
- Littlewood & Bergmann (1956)
- Ni (1972)

Unlikely Scalar-Tensor Theories

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | **Einstein 1915**

Whitehead 1922 | **Cartan 1923** | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | **Thiry 1948** | Papapetrou 1954 | **Jordan 1955** | Littlewood & Bergmann 1956

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Bollini et al. 1970 | Rosen 1971 | **Will & Nordtvedt 1972** | Ni 1972 | **Hellings & Nordtvedt 1972**

Ni 1973 | **Yilmaz 1973** | **Lightman & Lee 1973** | **Lee, Lightman & Ni 1974** | **Rosen 1975**

Belinfante & Swihart 1975 | **Lee et al. 1976** | **Bekenstein 1977** | **Barker 1978** | **Rastall 1979**

Coleman 1983 | **Logunov 1987** | **Hehl 1997** | **Overlooked (20th century)**

Scalar-Tensor theories are extremely constrained by Viking (1976) result on γ:

- Thiry (1948), Jordan 1955
- Brans & Dicke (1961): $\omega > 6500$
- Bergmann (1968), Nordtvedt (1970)
- Wagoner (1970), Bekenstein (1977)
- Barker (1978)

Perihelion Mercury: Theories that fail

Newton 1686 Poincaré 1890
Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**
Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943
Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956
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Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**
Ni 1973 **Yilmaz 1973** **Lightman & Lee 1973** **Lee, Lightman & Ni 1974** **Rosen 1975**
Belinfante & Swihart 1975 **Lee et al. 1976** Bekenstein 1977 Barker 1978 **Rastall 1979**
Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20th century)**

Stratified theories predict preferred frame effects on perihelion shift:

- Ni (1973)
- Lee, Lightman & Ni (1974)

GW & Binary Pulsar: Theories that fail

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

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Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20thcentury)**

Bi-metric Theories predict a dipole radiation. Can't be…:

- Rosen (1975)
- Lee et al. (1976)
- Rastall (1979)
- Lightman & Lee (1973)

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Some Theories resist to fail…

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

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Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20thcentury)**

- Will and Nordtvedt (1972) and Hellings and Nordtvedt (1972) are vector-tensor theories. Can be only significant in high energy (e.g. Planck energy) regimes to be in accord with current precision of experiments
- Yilmaz (1973) was mathematically inconsistent, but now is fixed. Does not predict black holes
- Logunov (1987) bi-metric theory of massive gravity; does not predict black holes, however fits all the solar system and binary pulsar data
- Cartan (1923), Hehl (1997) introduces matter spin

"Aesthetics-Based" Conclusion for 20th Century

Newton 1686   Poincaré 1890
Einstein 1912   Nordstrøm 1912   Nordstrøm 1913   Einstein & Fokker 1914   **Einstein 1915**
Whitehead 1922   Cartan 1923   Kaluza & Klein 1932   Fierz & Pauli 1939   Birkhoff 1943
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Belinfante & Swihart 1975   Lee et al. 1976   Bekenstein 1977   Barker 1978   Rastall 1979
Coleman 1983   Logunov 1987   Hehl 1997   Overlooked (20thcentury)

– "[...] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. [...] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions."

  – Malcolm MacCallum, 1976

– "Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity"

  – Misner, Thorne and Wheeler, 1973

35 Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Pioneers, Vikings, Cassini, Mars Global Surveyor, Mars Orbiter
- VLBI, GPS, etc.

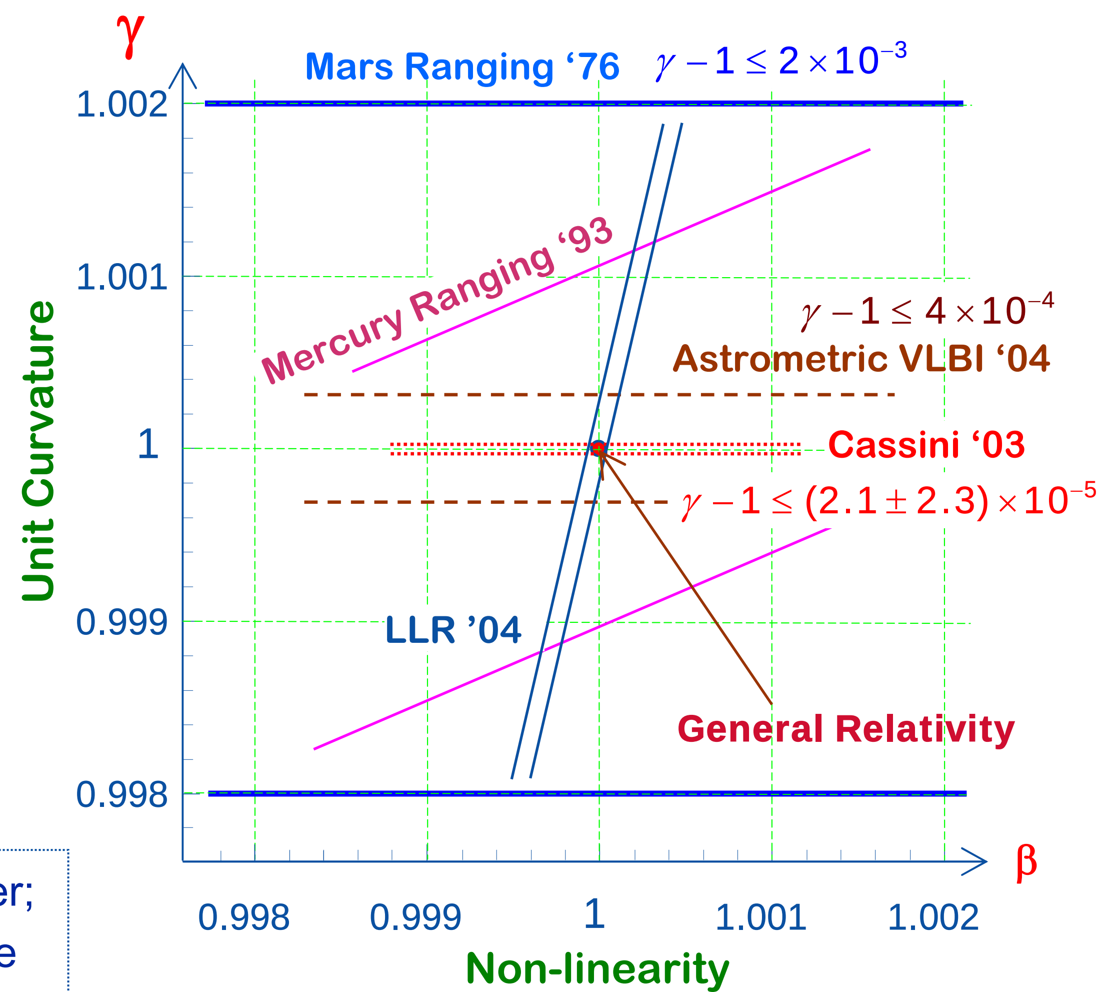
Laser:
- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '07; LISA, 2017

New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & astrometry (SIM, GAIA,....).



A factor of 100 in 35 years is impressive, but is not enough for the near future!

Theoretical Landscape of the 21st Century: *(Re-emergence of the)* Scalar Field

Fundamental Physics Challenges:

- Appearance of space-time singularities;
- Classical description breaks down in large curvature domains;
- Quest for Quantum Gravity $\rightarrow$ GR modification;
- Dark Energy…

Alternative Theories of Gravity:

- Grand Unification Models, Standard Model Extensions;
- Inflationary cosmologies, strings, Kaluza-Klein theories;

Common element: scalar partners – dilaton, moduli fields…

The scalar field is a pioneer, sent out to explore new worlds of physics!

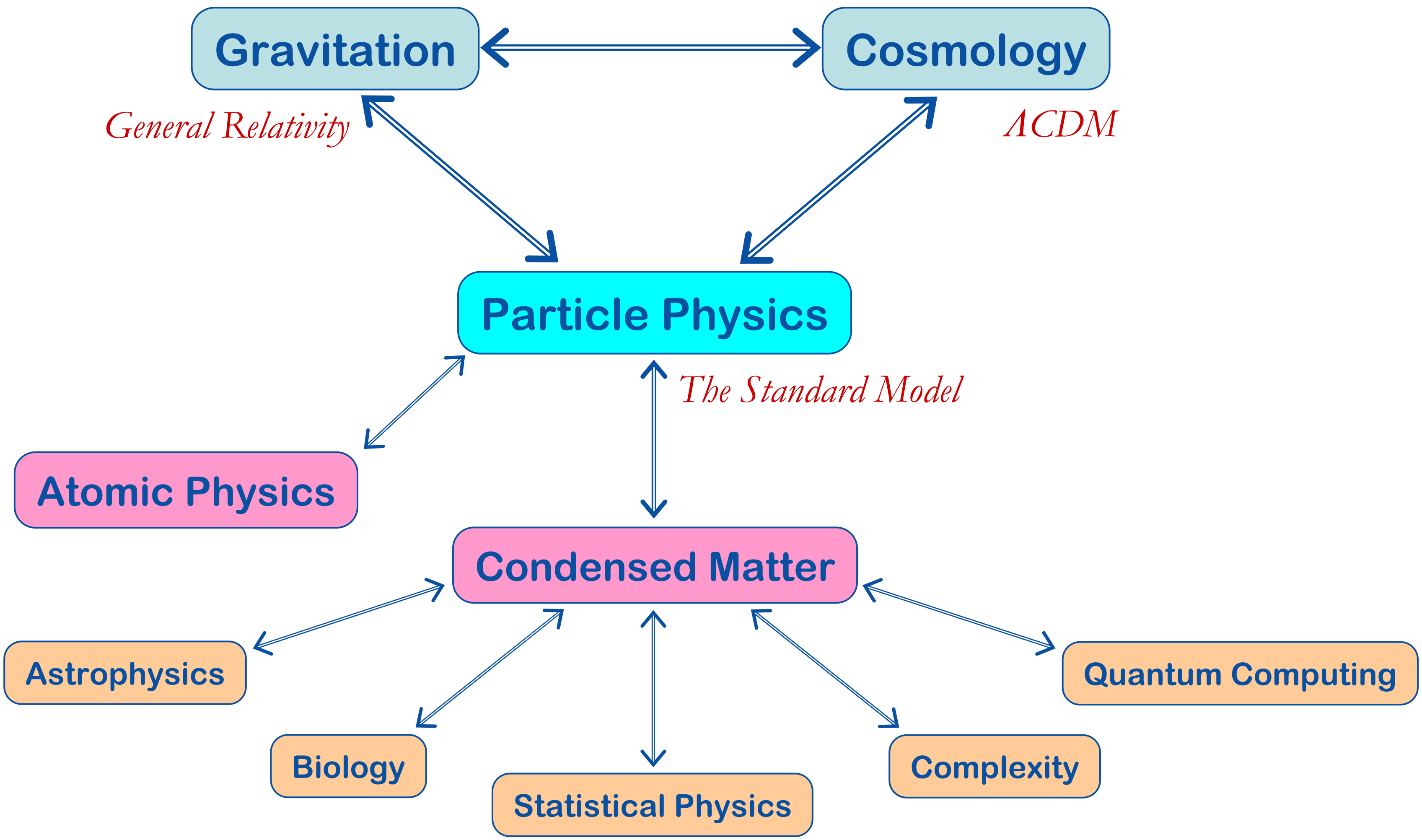
"Gravity and the Tenacious Scalar Field"
Carl Brans, gr-qc/9705069

- Waves, Optics
- Electrodynamics
- Quantum Mechanics
- Scalar QED
- Field Theory
- Symmetry Breaking
- Dilatons, Moduli
- …

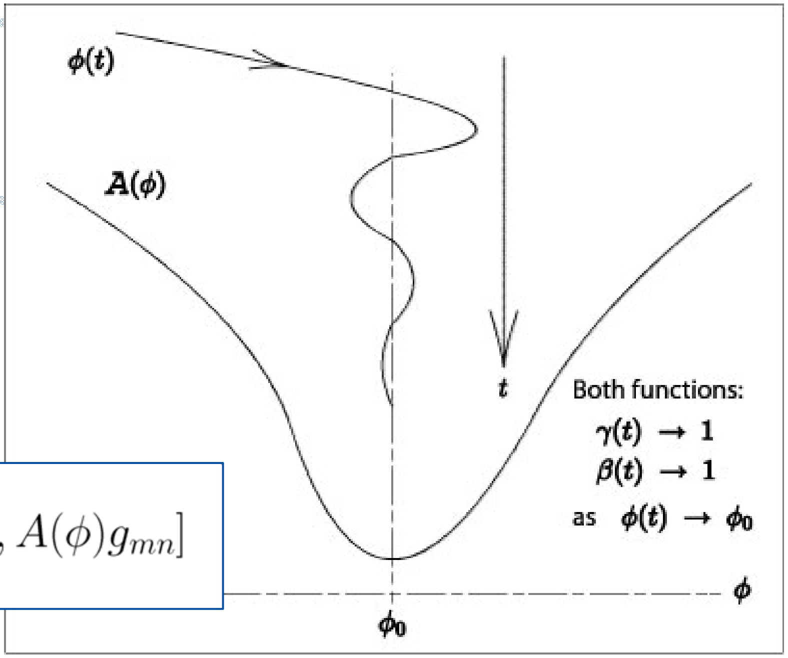
- Nordstrom's Scalar Gravity
- Kaluza-Klein Unification
- Dirac and Jordan's Cosmology
- Scalar-Tensor Gravity
- Inflation
- Quintessence
- TeVeS, STVG,
- …

Theoretical Landscape of the 21th Century:

Fundamental Physics in 21st century



Theoretical Motivation for New Gravity Tests



Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in 'Einstein Frame' (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G}\int dx^4\sqrt{-g}\Big(R - 2g^{mn}\nabla_m\phi\nabla_n\phi\Big) + S_M[\psi_M, A(\phi)g_{mn}]$$

Expansion *A(ϕ)* around background value ϕ_0 of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi - \varphi_0) + \frac{1}{2}k_0(\varphi - \varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope α_0 measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 = \frac{-2\alpha_0^2}{1+\alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2}\frac{\alpha_0^2 k_0}{(1+\alpha_0^2)^2} \simeq \frac{1}{2}\alpha_0^2 k_0 \simeq \frac{1}{4}(1-\gamma)k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

$$\gamma - 1 \sim 7.3\times 10^{-7}\left(\frac{H_0}{\Omega_0^3}\right)^{\frac{1}{2}} \quad \Rightarrow \quad \gamma - 1 \sim 10^{-5} - 10^{-7}$$

The unit curvature, PPN parameter γ – the most important quantity to test

– General relativity (GR): $$S = \int d^4x \sqrt{g} \left(-\frac{1}{16\pi G} R + \mathcal{L}_m \right)$$

$$\delta S = \int d^4x \sqrt{g} \left(-\frac{1}{16\pi G} \right) \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G T_{\mu\nu} \right) \delta g^{\mu\nu} = 0$$

– Brans-Dicke, generalization of GR where *G* is no longer a constant:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{g} \left[-\phi R + \frac{\omega(\phi)}{\phi} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \qquad (1)$$

$$G_{\text{eff}} \simeq \phi^{-1}$$

– Conformal transformation: $g_{\mu\nu} \to \phi^{-1} g_{\mu\nu}$

$$\Rightarrow \quad S = \int d^4x \sqrt{g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

– Scalar-tensor theories of the form (1) naturally arise from e.g. extra dimensions

Dark Energy - Matter Coupling — Amendola et al, ApJ 583 (2003) L53
Dark Energy Solid — Bucher & Spergel, PRD 60 (1999) 043505
Chaplygin Gas — Kamenshchik et al, PLB 511 (2001) 265
Bulk Pressure of Particle Creation — Zimdahl, PRD 61 (2000) 083511
Quantum Effects of a Scalar Field — Parker & Vanzella, PRD 69 (2004) 104009
…

Dark Energy & Gravity: Non-Minimal Coupling

Scalar-Tensor Models

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\partial\phi^2 - V(\phi)\right] + S_m[g_{\mu\nu},\psi]$$

GR as an attractor of S-T Theories:
Damour & Nordtvedt, PRL 70 (1993) 2217

Perrotta et al, PRD 61 (2000) 023507
Bartolo et al, PRD 61 (2000) 023518
…

Modifications of Einstein Gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu},\psi]$$

Carroll et al, PRD 70 (2004) 043528
…

Modification of Einstein Gravity

Olmo, gr-qc/0505135,6

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}F(R) + S_m[g_{\mu\nu},\,\psi]$$

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,[F(A) + (R-A)B] + S_m[g_{\mu\nu},\,\psi]$$

$$\varphi = B,\; V(B) = AB - F(A)$$

Metric Variation of the Action

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\varphi R - \frac{\omega(\varphi)}{\varphi}\partial\varphi^2 - V(\varphi)\right] + S_m[g_{\mu\nu},\,\psi]$$

$$\omega = 0$$

Palatini Variation of the Action

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\varphi R - \frac{\omega(\varphi)}{\varphi}\partial\varphi^2 - V(\varphi)\right] + S_m[g_{\mu\nu},\,\psi]$$

$$\omega = -3/2$$

The scalar field is non-dynamical. Instead, it is a derived quantity which depends on the local matter density and the potential.

Modification of Einstein Gravity

Olmo, gr-qc/0505135,6

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}F(R) + S_m[g_{\mu\nu},\,\psi]$$

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,[F(A) + (R-A)B] + S_m[g_{\mu\nu},\,\psi]$$

$$\varphi = B,\; V(B) = AB - F(A)$$

Metric Variation of the Action

- $$S = \int d^4x\sqrt{-g}\frac{1}{16\pi G_N}\left[R - \frac{\mu^4}{R}\right],$$

(Capozziello et al astro-ph/0303041, Carroll et al astro-ph/0306438)

- $$S = \int d^4x\sqrt{-g}\frac{1}{16\pi G_N}\left[R - \frac{\mu^{4n+2}}{(aR^2 + bP + cQ)^n}\right],$$

(Carroll et al astro-ph/0410031, fit to the SN data: Mena et al astro-ph/0510453)

Modification of PPN Gravity

$$\gamma - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2},$$

$$\beta - 1 = \frac{1}{4}\frac{f'(R) \cdot f''(R)}{2f'(R) + 3f''(R)^2}\frac{d\gamma}{dR}.$$

Analogy between scalar-tensor and higher-order gravity

Constraints on … $f(R)$

…tight restrictions on the form of the gravitational Lagrangian

Need for cosmological PPN formalism

Capozziello, Stabile, Trosi, gr-qc/0603071

Theory: Lorentz Invariance *violations*

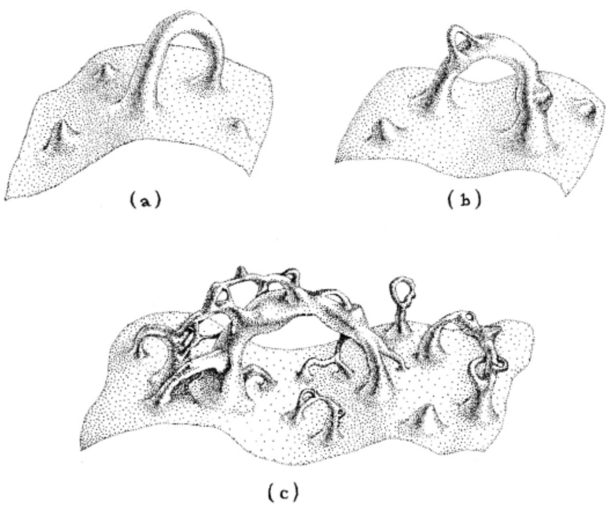
The idea of a smooth, continuous spacetime breaks down near the Planck scale

$$l_P = (\hbar G/c^3)^{1/2} \approx 10^{-33}\,\mathrm{cm}$$

in many theories of quantum gravitational phenomena. As a practical consequence, such theories predict violations of Lorentz Invariance in the form of the dispersion relation

$$E^2 = m^2 + p^2 \times \left[1 + f_n \left(\frac{p}{m_P}\right)^n\right]$$

overview: Mattingly gr-qc/0502097



Kip Thorne 1994 "Black holes and time machines: Einstein's outrageous legacy"

- Forbidden decays now allowed
- Relativistic γ-factor has different meaning
- Possible CPT violation?

numerous astrophysics, cosmology implications

Experiment: search for preferred-frame effects

frame1 : $S(T, X)$ e.g. CMB $v_{sol} \approx 377\,\text{km/s}$

frame2 : $s(t, x)$ laboratory $RA, dec = (11.2, -6.4^\circ)$

Mansouri & Sexl, 1977

$$dT = \frac{1}{a}(dt + \frac{v}{c^2}dx) \qquad a = 1 + \alpha\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx) \qquad b = 1 + \beta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dY = \frac{1}{d}dy,\; dZ = \frac{1}{d}dz \qquad d = 1 + \delta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

SR: $\alpha = -1/2,\ \beta = 1/2,\ \delta = 0$

Experiments:

$$P_{MM} = (\tfrac{1}{2} - \beta + \delta)$$ Michelson-Morley: orientation dependence

$$P_{KT} = (\beta - \alpha - 1)$$ Kennedy-Thorndike: velocity dependence

$$P_{IS} = |\alpha + \tfrac{1}{2}|$$ Ives-Stillwell: contraction, dilation

$$P_{MM} = -0.9(\pm 2.0) \times 10^{-10}$$

Stanwix et al, PRL 95 (2005) 040404

$$P_{KT} = 3.1(\pm 6.9) \times 10^{-7}$$

Wolf et al, PRL 90 (2003) 060402

$$P_{IS} < 2.2 \times 10^{-7}$$

Saathoff et al, PRL 91 (2003) 190403

Precision tests of Lorentz Invariance

Experiments:

$$\Delta c_\theta / c = 2.6(\pm 1.7) \times 10^{-15}$$

isotropy of the speed of light

Muller et al, PRL 91 (2003) 020401

Standard Model Extensions (SME)

overview: Bluhm hep-ph/0506054

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} (k_F)_{\lambda\kappa\mu\nu} F^{\lambda\kappa} F^{\mu\nu}$$

$$k_F < 10^{-11},\ 10^{-15}$$

Muller et al, PRL 91 (2003) 020401

$$k_F < 10^{-31}$$ (astrophysical)

Kostelecky et al, PRL 87 (2001) 251304

$$\delta H_n < 10^{-27}\ \mathrm{GeV}$$

boost-invariance of neutron

Cane et al, PRL 93 (2004) 230801

Theoretical Landscape of the 21th Century:

Competing Theories of Gravity

7 years later … – they are back!

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 **Kaluza & Klein 1932** Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Logunov 1987 Hehl 1997 Overlooked (20th century) **Scalar-Tensor Theories**

Arkani-Hamed, Dimopoulos & Dvali 2000 **Dvali, Gabadadze & Poratti 2003** **Strings theory?**

Bekenstein 2004 **Moffat 2005** **Multiple F(R) models 2003-07** **Bi-Metric Theories**

Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

Other challenges:

- Dark Matter
- Dark Energy
- Pioneer Anomaly…

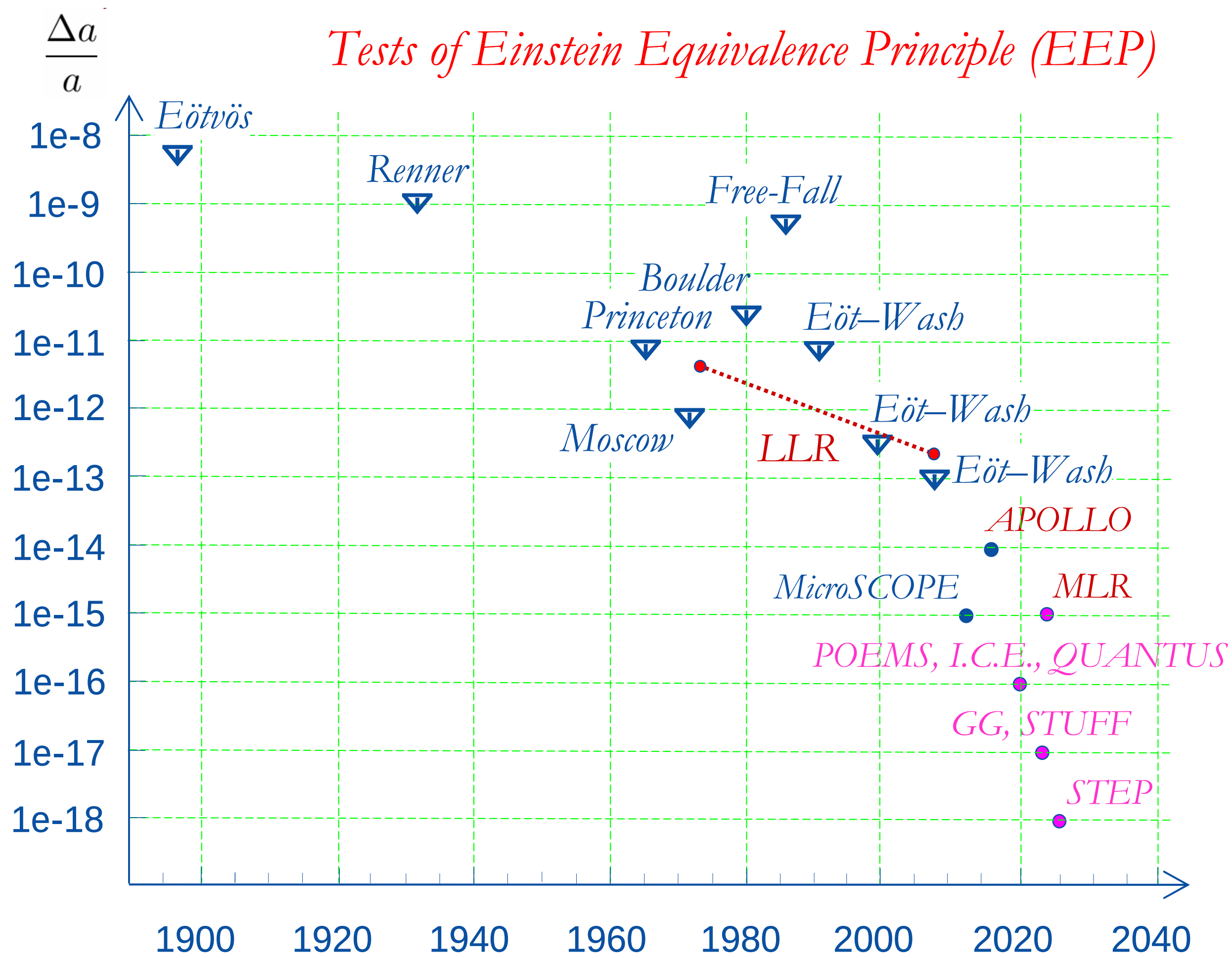
Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment

Tests of Einstein Equivalence Principle (EEP)



- • funded projects
- • proposed projects
- *LLR*, *APOLLO*, and *MLR* testing the Strong Equivalence Principle (SEP)

Uniqueness of Free Fall

(≡ Weak Equivalence Principle):

$$\vec{F} = m_I \vec{a} = m_G \vec{g}$$
$$\Rightarrow m_I = m_G$$

All bodies fall with the same acceleration

Define the test parameter that signifies a violation of the **WEP**

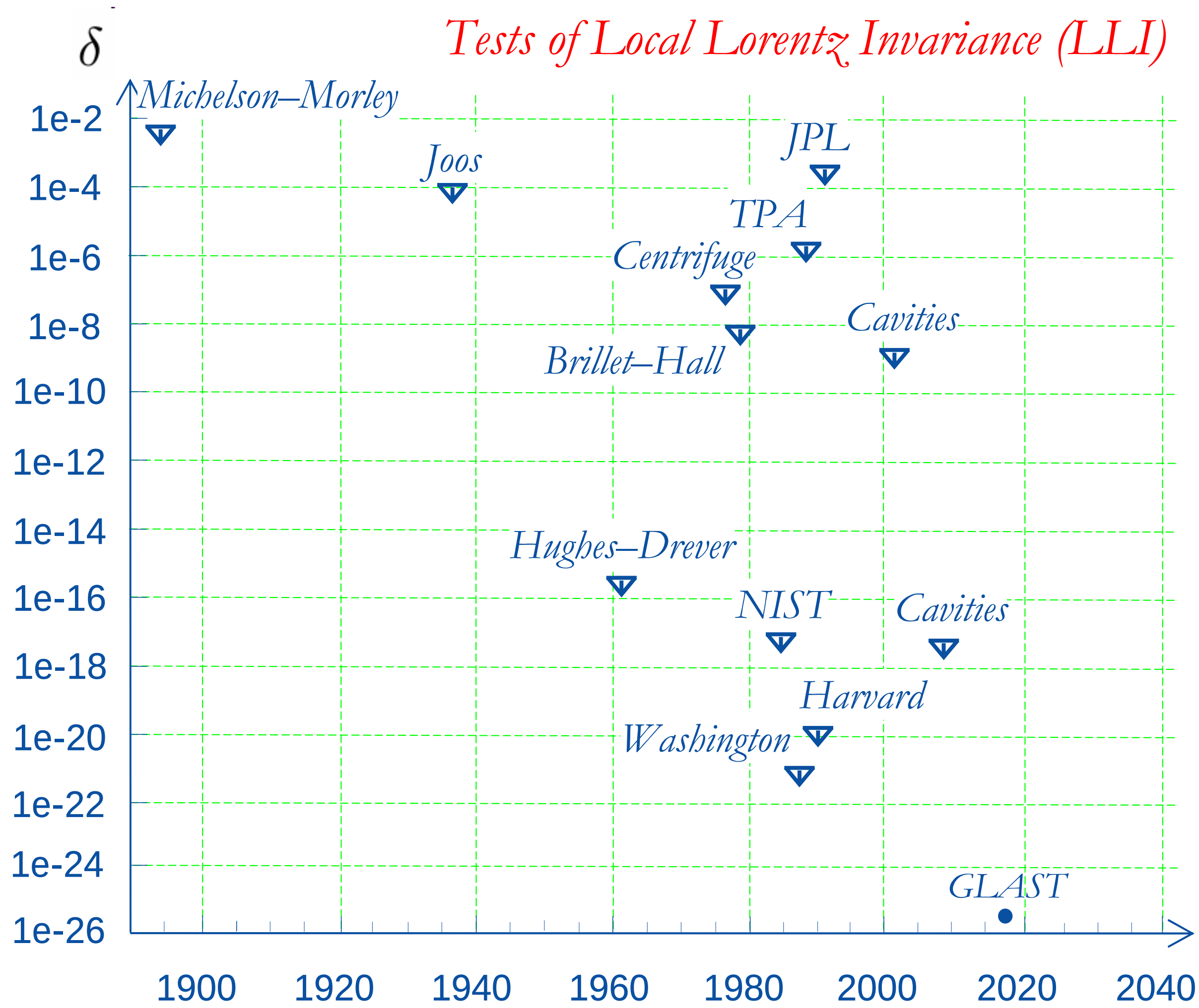
$$\frac{\Delta a}{a} = \frac{(a_1 - a_2)}{\frac{1}{2}(a_1 + a_2)} = \left[\frac{m_G}{m_I}\right]_1 - \left[\frac{m_G}{m_I}\right]_2$$

Let Ω is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

$$\frac{\Delta a}{a} = (4\beta - \gamma - 3)\left\{\left[\frac{\Omega}{mc^2}\right]_1 - \left[\frac{\Omega}{mc^2}\right]_2\right\}$$

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



– Michelson-Morley, Joos, Brillet-Hall: Round-trip propagation

– Centrifuge, TPA, JPL: One-way propagation

– Rest: Hughes-Drever experiments

$$\delta \equiv \frac{c^2}{c_0^2} - 1$$

Local Lorentz Invariance:

– Extended frameworks by Kostelecky et al., Jacobson et al.

Future experiments:

– Clock comparisons

– Clocks vs cavities

– Time of flight of high energy photons

– Birefringence in vacuum

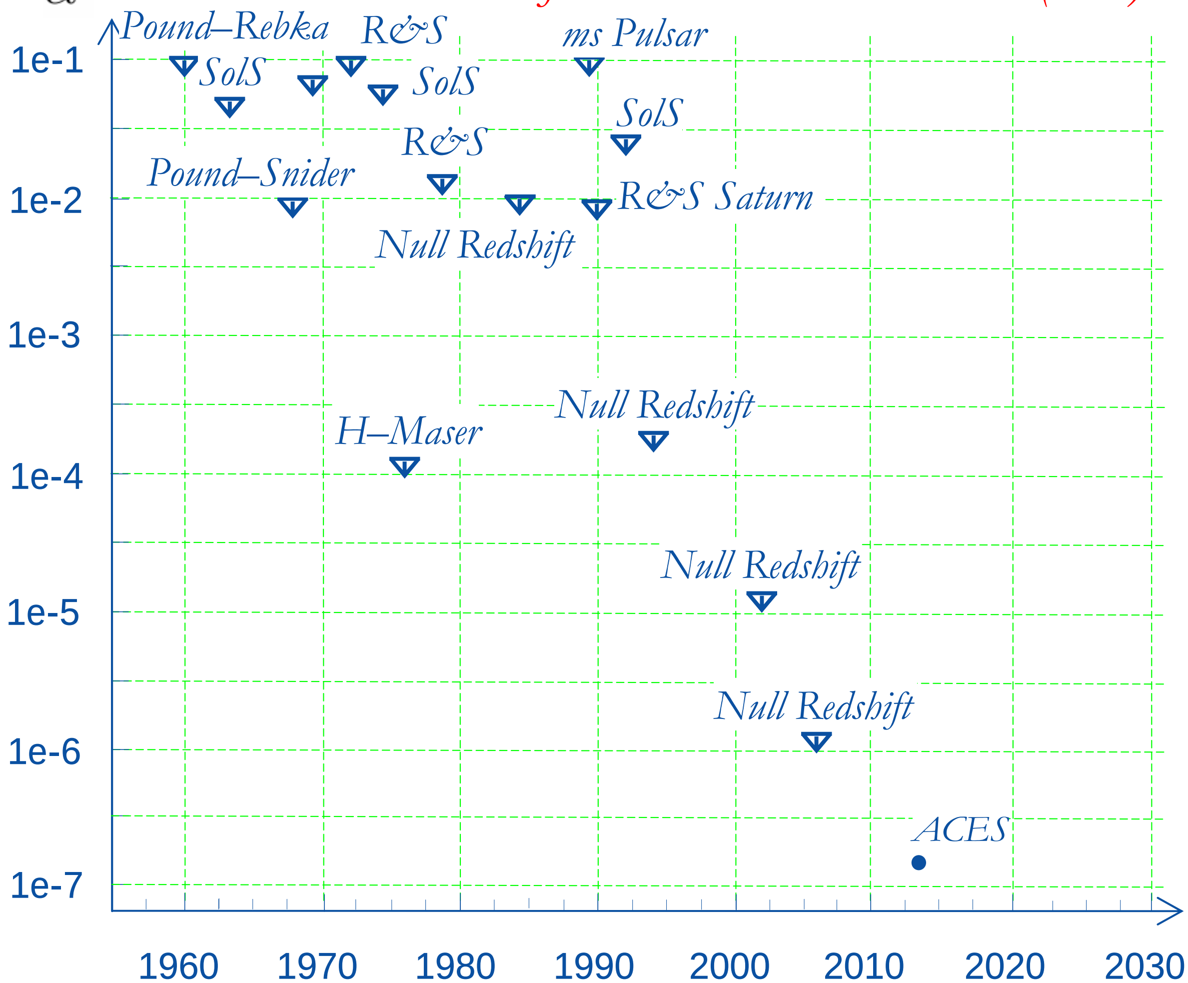
– Neutrino oscillations

– Threshold effects in particle physics

Test of one-way speed of light:

– Important to fundamental physics, cosmology, astronomy and astrophysics

Tests of Local Position Invariance (LPI)



- SolS: Solar Spectra
- R&S: Rockets and Spacecraft
- Null Redshift: comparison of different atomic clocks

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

Local Position Invariance:

- The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

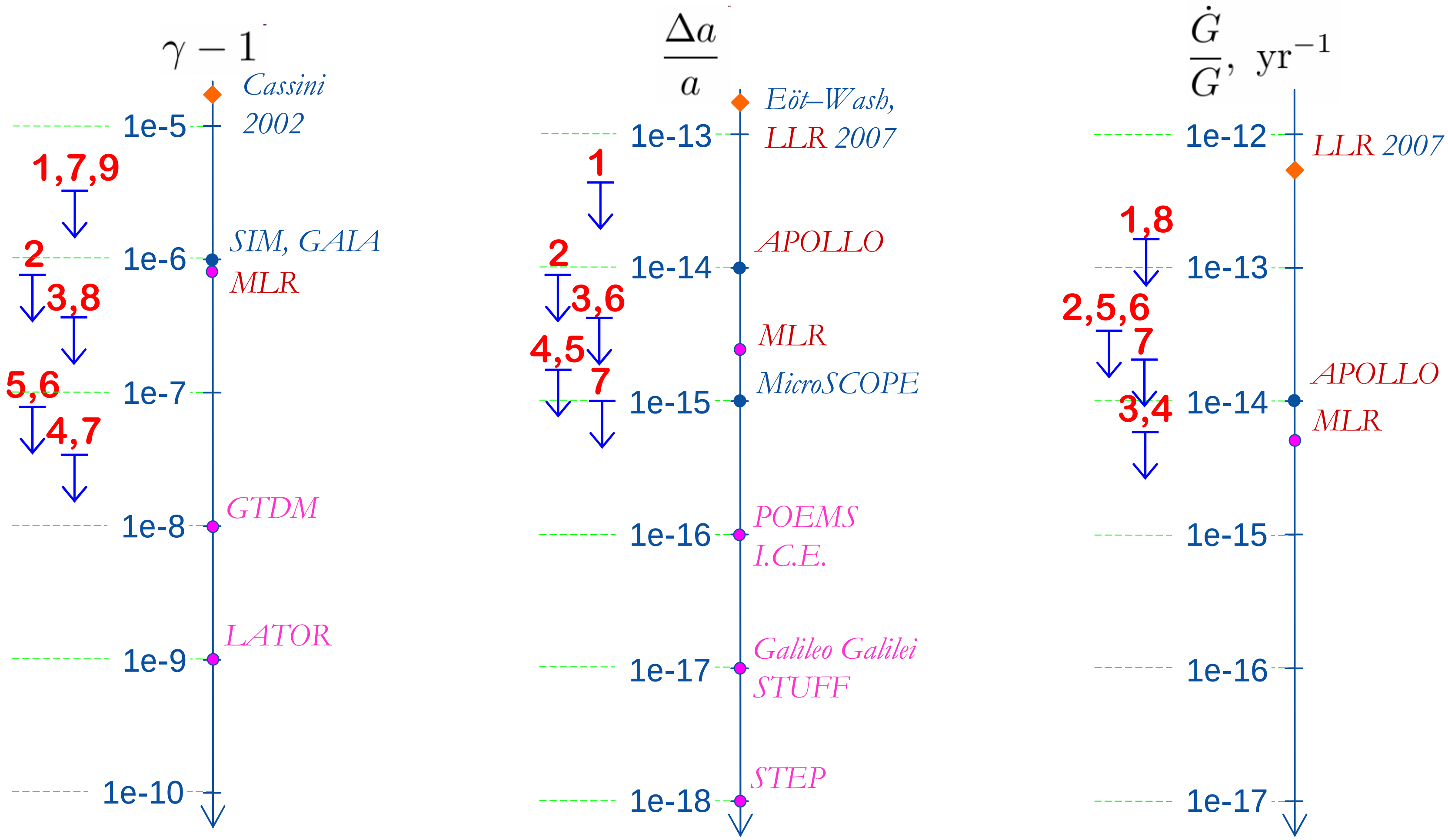
Splits into:

- spatial invariance
- temporal invariance
- Current best result is by Ashby et a., Phys. Rev. Lett. 98, 070802 (2007)

$$|\alpha| < 1.4 \times 10^{-6}$$

Theoretical Landscape of the 21th Century:

Confrontation Between Theory and Experiment



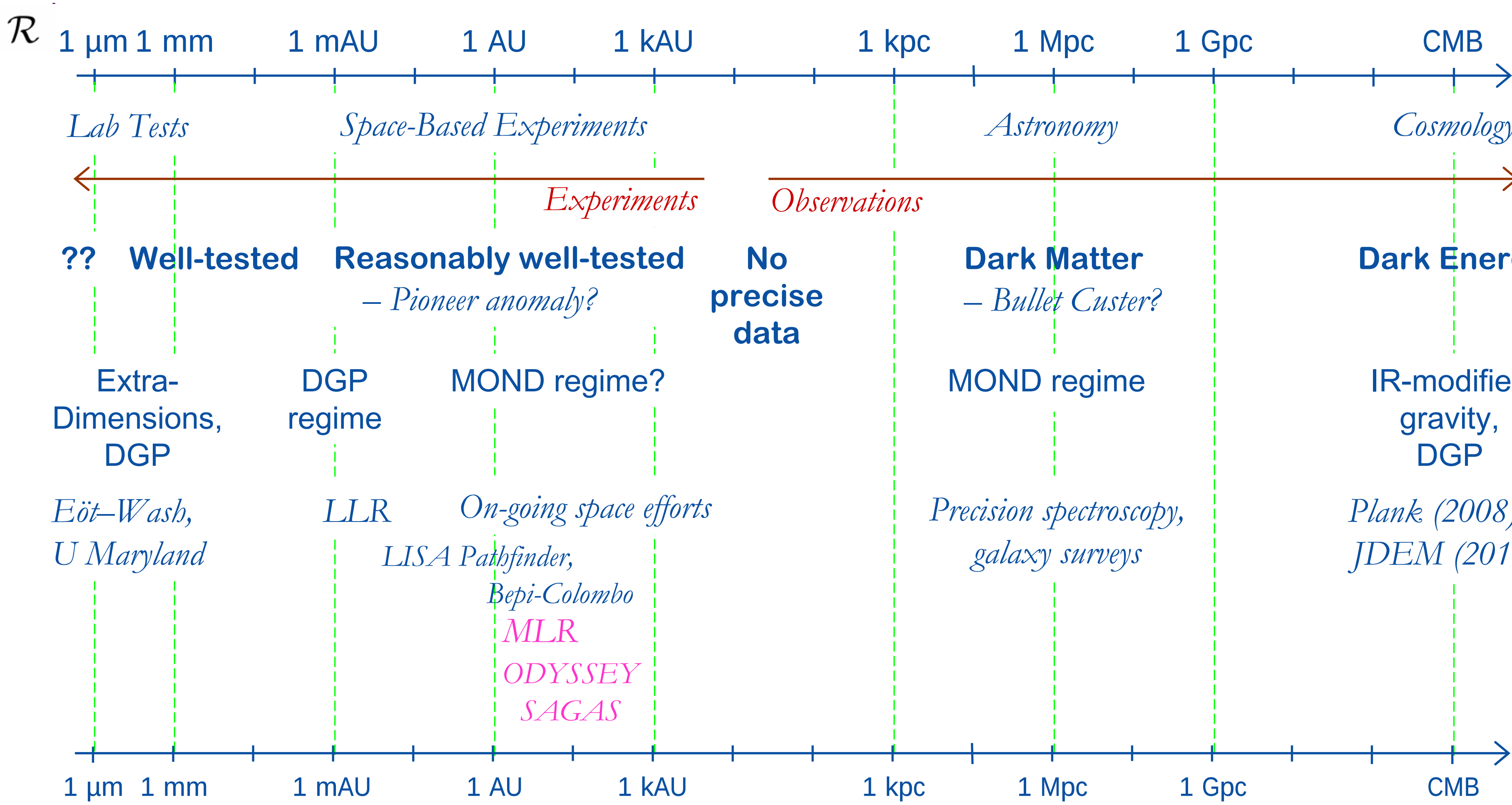
New Theories & Future Tests

1 *Damour-Polyakov-Nordtvedt 1993*
2 *Damour-Esposito-Farese 1996*
3 *Damour-Piazza-Veneziano 2002*
4 *Arkani-Dimopoulos-Dvali 2000*
5 *Dvali-Gabadadze-Poratti 2003*
6 *F(R) gravity models 2003-07*
7 *Bekenstein 2004*
8 *Moffat 2005*
9 *Jaekel-Reynaud 2006*

◆ *Current best* ● *Funded* ● *Proposed*

Theoretical Landscape of the 21th Century:

How well to we know gravity?

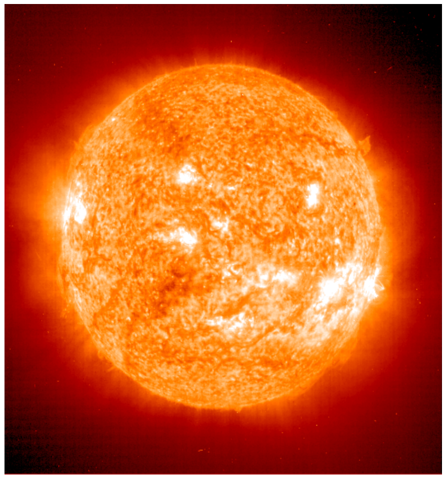


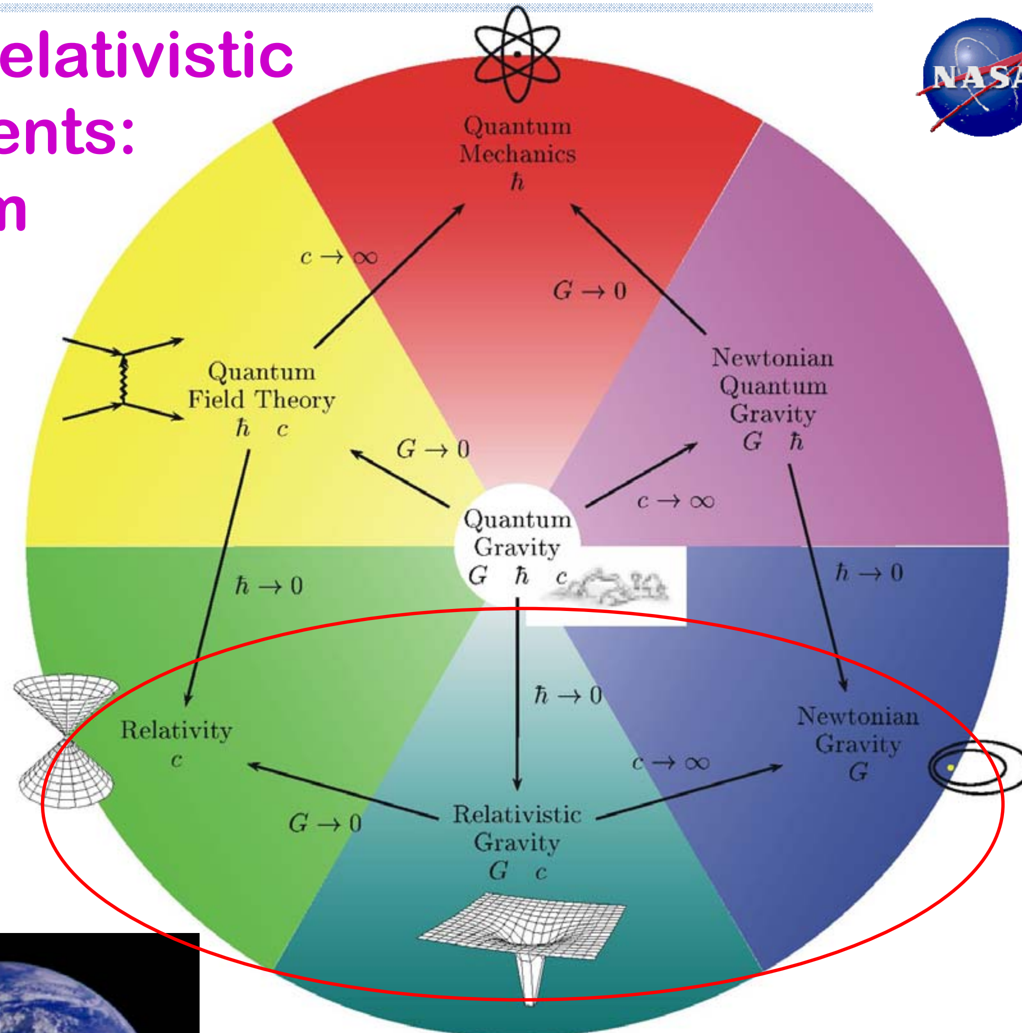
New Theories & Future Tests: All theories and models are relevant for all regimes

1 *Damour-Polyakov-Nordtvedt 1993*
2 *Damour-Esposito-Farese 1996*
3 *Damour-Piazza-Veneziano 2002*
4 *Arkani-Dimopoulos-Dvali 2000*
5 *Dvali-Gabadadze-Poratti 2003*
6 *F(R) gravity models 2003-07*
7 *Bekenstein 2004*
8 *Moffat 2005*
9 *Jaekel-Reynaud 2006*

• *Funded*
• *Proposed*

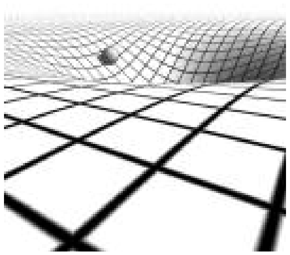
Laboratory for Relativistic Gravity Experiments: Our Solar System





Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$



$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$

Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

Technology is available to conduct tests in immediate solar proximity

Deep Space Network

Goldstone, California

Goldstone, California

Canberra, Australia

Madrid, Spain

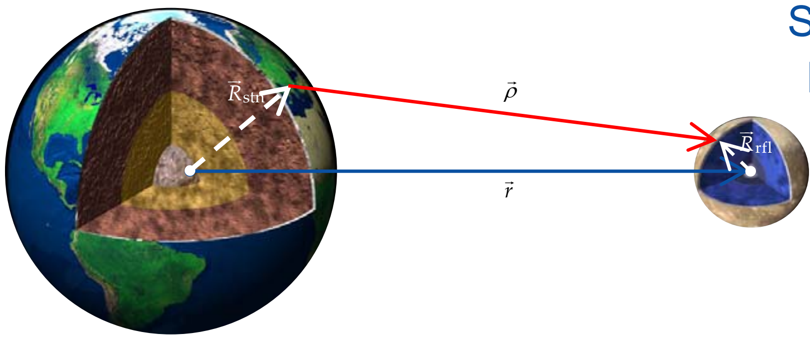
Navigation Tracking Requirements (2007)

*Based on the current (2007) set of anticipated missions

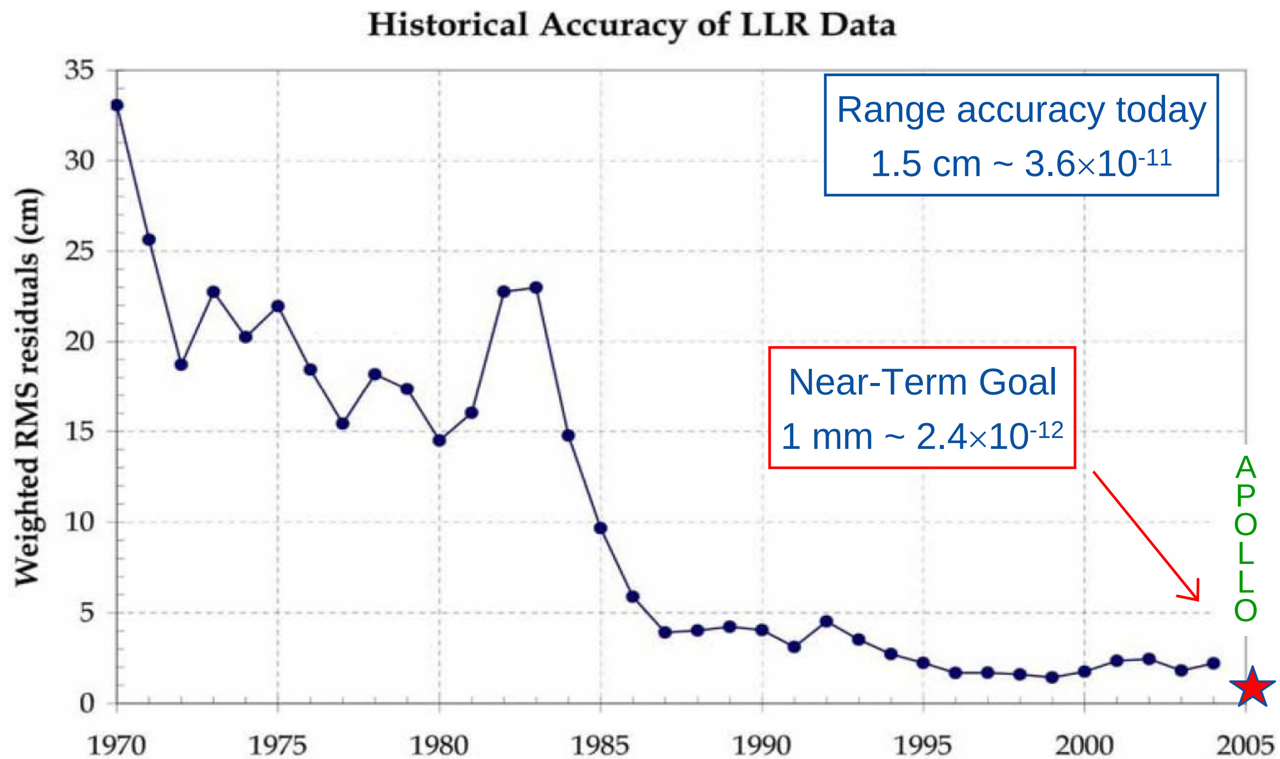
Tracking Error Source (1σ Accuracy)	Units	current capability	2010 reqt*	2020 reqt*	2030 reqt*
Doppler/random (60s)	μm/s	30	30	30	20
Doppler/systematic (60s)	μm/s	1	3	3	2
Range/random	m	0.3	0.5	0.3	0.1
Range/systematic	m	1.1	2	2	1
Angles	deg	0.01	0.04	0.04	0.04
ΔVLBI	nrad	2.5	2	1	0.5
Troposphere zenith delay	cm	0.8	0.5	0.5	0.3
Ionosphere	TECU	5	5	3	2
Earth orientation (real-time)	cm	7	5	3	2
Earth orientation (after update)	cm	5	3	2	0.5
Station locations (geocentric)	cm	3	2	2	1
Quasar coordinates	nrad	1	1	1	0.5
Mars ephemeris	nrad	2	3	2	1

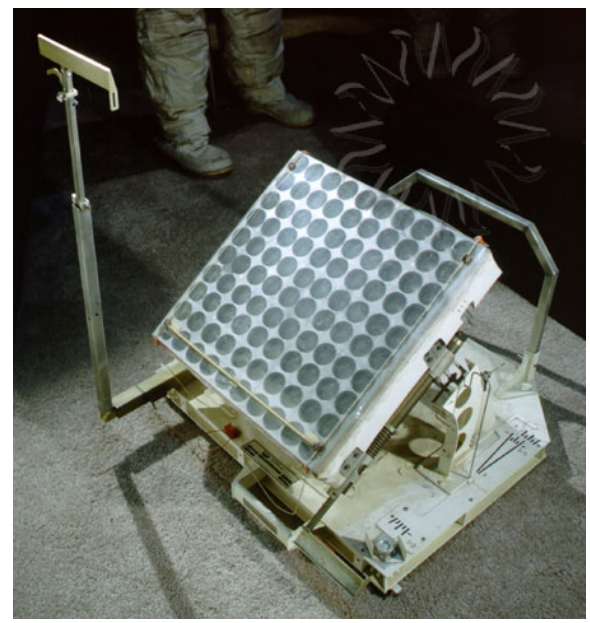
Interplanetary laser ranging is a very natural step to improve the accuracy

Lunar Laser Ranging



Schematics of the LLR experiment





LLR contributes to astrometry, geodesy, geophysics, lunar planetology, gravitational physics

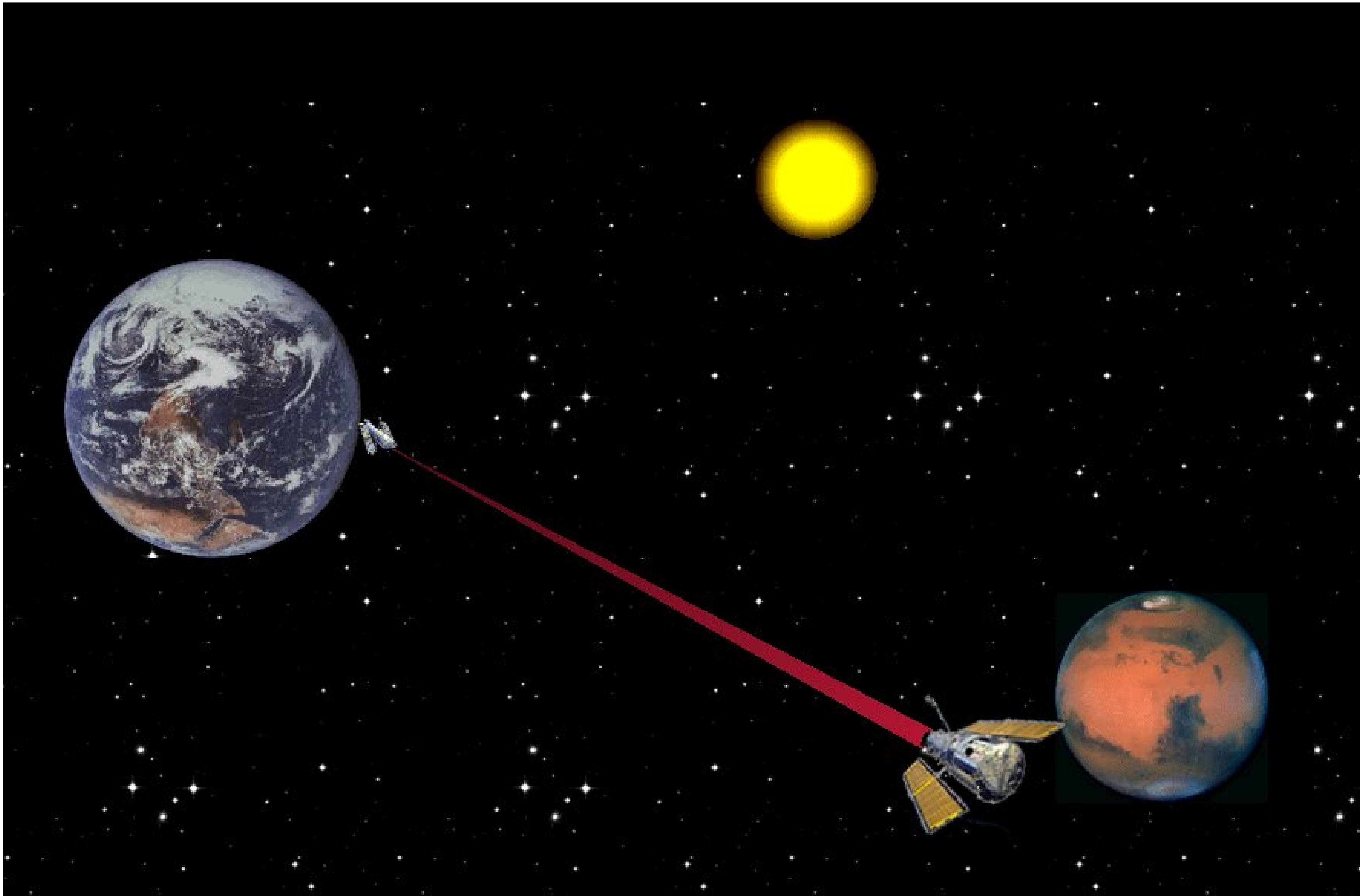
Expected Benefits from Next Generation of LLR

Lunar science effects that will benefit from a wider distribution of LLR arrays

Effect	Current	Future Goals
Positions on Moon	yes	More locations
Low-degree gravity field	yes	Distinguish mantle from inner core for gravity and moments
3 free libration mantle modes	yes	Seek stimulating events
Solid-body tides	yes	Improve Love number accuracies
Tidal dissipation	yes	Improve tidal Q vs frequency
Core/mantle boundary dissipation	yes	Improve uncertainty, used to limit fluid core size
Core/mantle boundary flattening	yes	Improve uncertainty
Fluid core moment of inertia	no	Detect and determine
Fluid core free precession mode	no	Detect mode, determine amplitude & period
Inner solid core	no	Detect inner core, determine gravity
3 inner core free libration modes	no	Detect modes, determine amplitudes & periods
Inner core boundary dissipation	no	Limit inner core size

Earth geodesy and geophysics effects that would benefit from a bright lunar target (i.e., laser transponder).

Effect	Current	Future Goals
Station positions & motions	yes	More stations on more plates
UT and polar motion	yes	More stations, improved UT1 and polar motion accuracy including diurnal and semidiurnal variations
Precession and nutation	yes	Improved accuracy
Obliquity and equinox	yes	Improved accuracy



Next Step – Interplanetary Laser Ranging

Summary of Recent Transponder Experiments

Experiment	MLA (cruise)		MOLA (Mars)
Range (10^6 km)	24.3		~80.0
Wavelength, nm	1064		1064
	Uplink	Downlink	Uplink
Pulsewidth, nsec	10	6	5
Pulse Energy, mJ	16	20	150
Repetition Rate, Hz	240	8	56
Laser Power, W	3.84	0.16	8.4
Full Divergence, μrad	60	100	50
Receive Area, m^2	0.042	1.003	0.196
EA-Product, J-m^2	0.00067	0.020	.0294
PA-Product, W-m^2	0.161	0.160	1.64

- Key instrument parameters for recent deep space transponder experiments
- Note, these were experiments of opportunity and not design
- At the same time, the accuracy of MLA range determination was 12 cm at the distance of 24 mln km from the Earth (Sun et al., 2005, Smith et al., 2005)

Comparison of Laser-Enabled Gravity Tests

Relativistic Effect	LLR current	APOLLO	1 cm range to Mars	Combined LLR & Mars
Tests of the Equivalence Principle				
Weak Equivalence Principle, $\Delta a/a$	1.9×10^{-13}	1×10^{-14}	3×10^{-15}	3×10^{-15}
Strong Equivalence Principle, η	4.3×10^{-4}	2×10^{-5}	2×10^{-6}	2×10^{-6}
Determination of the PPN parameter β	1.1×10^{-4}	7×10^{-6}	1×10^{-6}	1×10^{-6}
Determination of the PPN parameter γ	2×10^{-3}	1×10^{-3}	1×10^{-6}	1×10^{-6}
Limits on the time variation of the gravitational constant G, $\dot{G}/G$	6×10^{-13} yr^{-1}	1×10^{-14} yr^{-1}	1×10^{-14} yr^{-1} asteroids…	7×10^{-15} yr^{-1} asteroids…
Gravitational inverse square law (testing for new long range forces)	3×10^{-10} at 4×10^{6} km	3×10^{-11} at 4×10^{6} km	3×10^{-11} at 2 AU	1×10^{-11} at 0.1-2 AU
Relativistic geodetic precession	4.7×10^{-3} lunar orbit	3×10^{-4} lunar orbit	3×10^{-4} Martian orbit	3×10^{-4} both lunar & Martian orbits

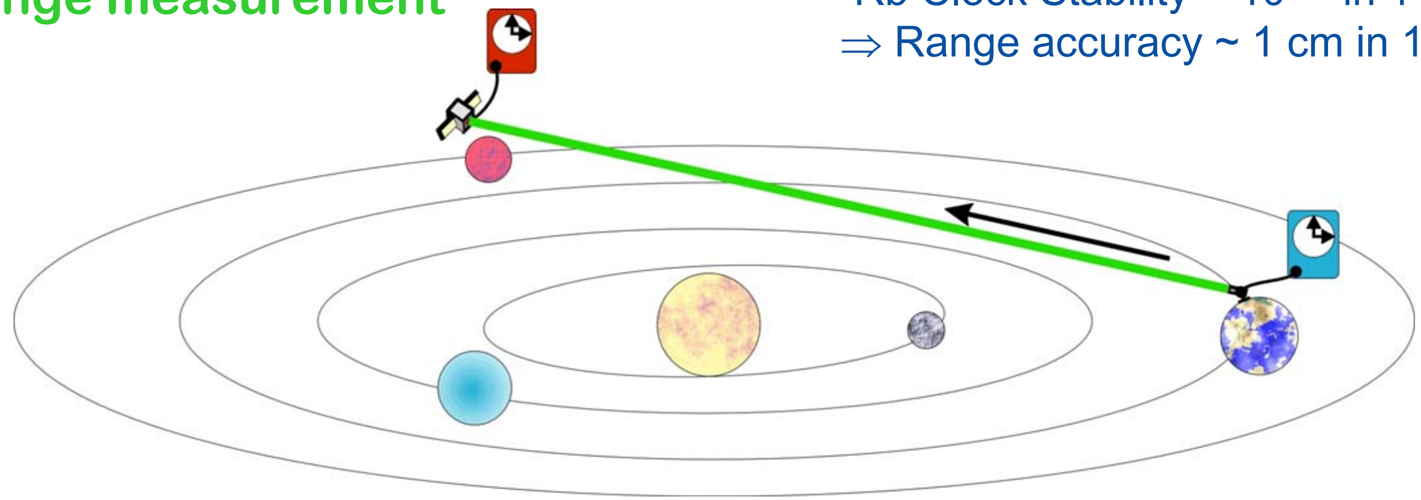
Numbers extrapolated from references below (we need a detailed covariance study):

J.D. Anderson, M. Gross, K.L. Nordtvedt, S.G. Turyshev, ApJ, 459 (1996) 365-370 [arXiv:gr-qc/9510029]
J.G. Williams, S.G Turyshev, D.H. Boggs, Phys.Rev.Lett.93:261101 (2004) [arXiv:gr-qc/0411113]
J.F. Chandler, M.R. Pearlman, R.D. Reasenberg, J.J. Degnan, in Proc. 14th LSRS Meeting, 2004
S.G Turyshev, J.G. Williams, [arXiv:gr-qc/0611095]

Next Step – Interplanetary

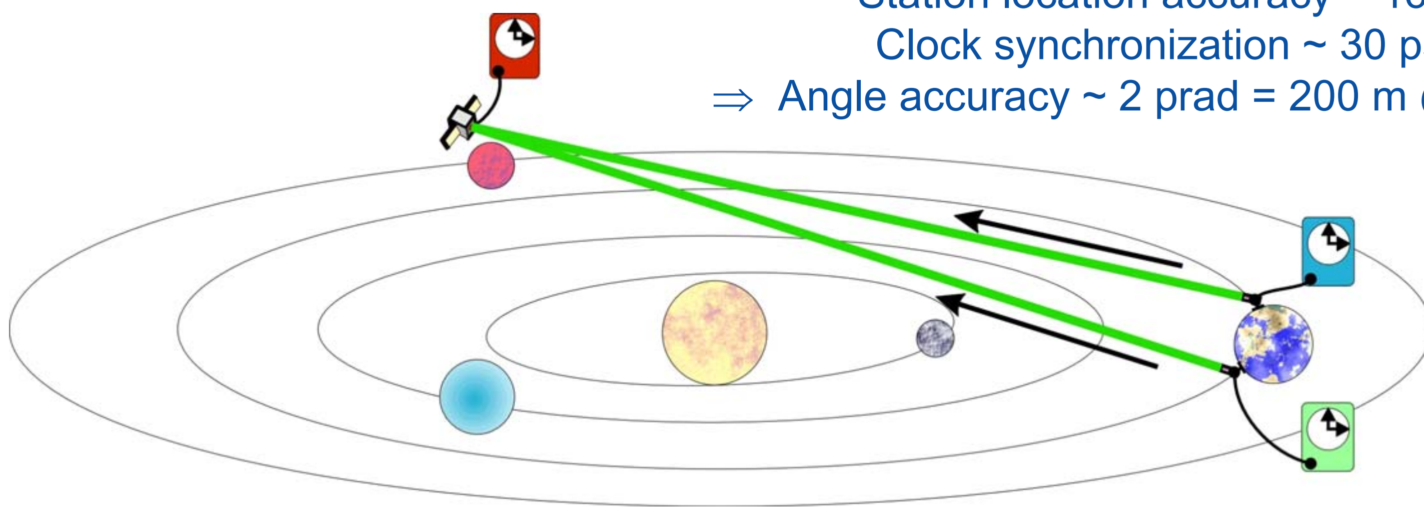
TIPO – range measurement

Rb Clock Stability ~ 10^{-15} in 1 hour
⇒ Range accuracy ~ 1 cm in 1 hour



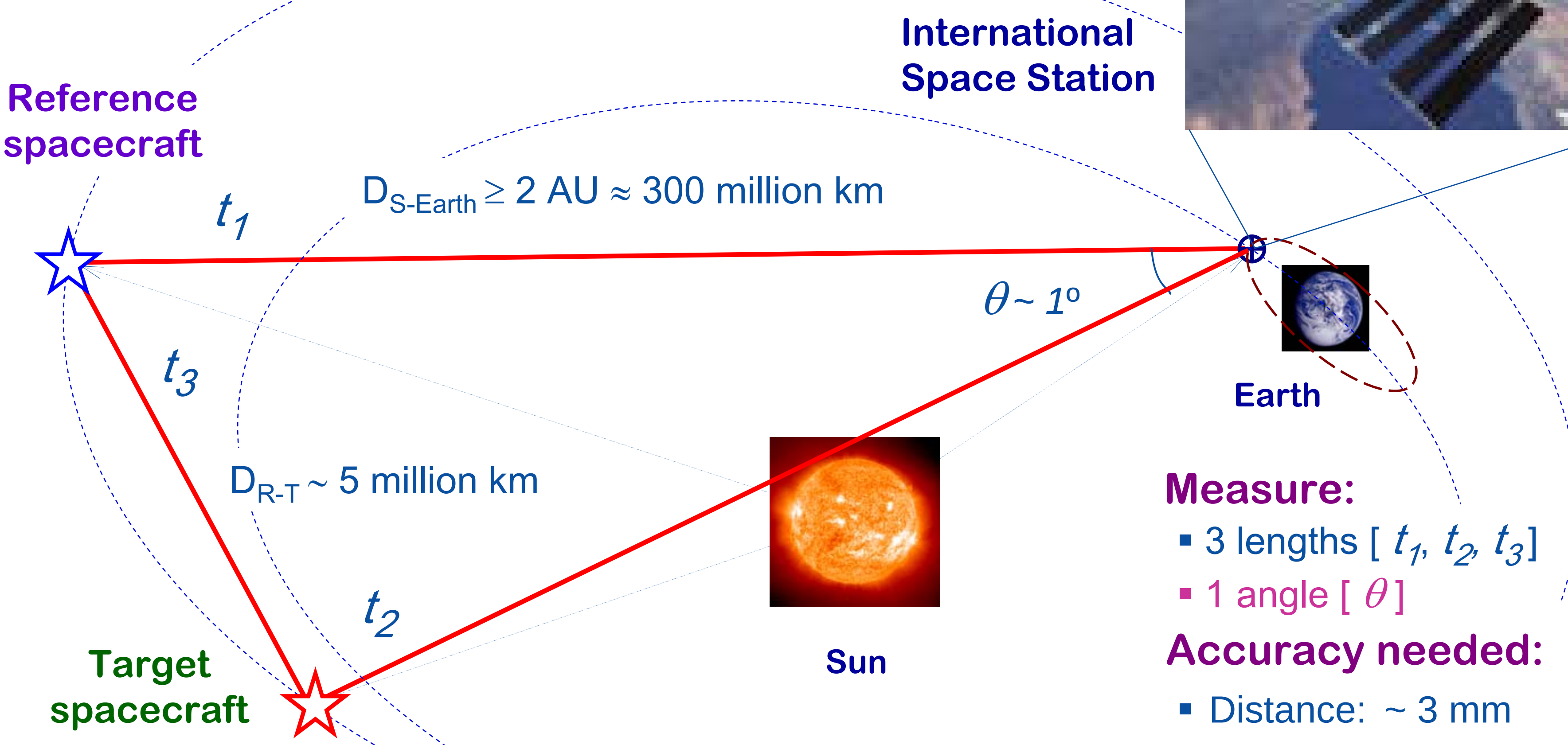
TIPO – angle measurement

Intercontinental baseline length ~ 10,000 km
Station location accuracy ~ 1cm
Clock synchronization ~ 30 ps
⇒ Angle accuracy ~ 2 prad = 200 m @ 10^8 km



LATOR Mission Concept

CQG 21 (2004) 2773-2799, gr-qc/0311020



Measure:

- 3 lengths [t_1, t_2, t_3]
- 1 angle [θ]

Accuracy needed:

- Distance: ~ 3 mm
- Angle: 0.01 picorad

Euclid is violated in gravity:

$$\cos\theta \neq (t_1^2 + t_2^2 - t_3^2)/2t_1t_2$$

Geometric redundancy enables a very accurate measurement of curvature of the solar gravity field

Accurate test of gravitational deflection of light to 1 part in 10^9

Thank You!